

# Topological Superfluid in one-dimensional Ultracold Atomic System with Spin-Orbit Coupling

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We propose a one-dimensional Hamiltonian  $H_{1D}$  which supports Majorana fermions when  $d_{x^2-y^2}$ -wave superfluid appears in the ultracold atomic system and obtain the phase-separation diagrams both for the time-reversal-invariant case and time-reversal-symmetry-breaking case. From the phase-separation diagrams, we find that the single Majorana fermions exist in the topological superfluid region, and we can reach this region by tuning the chemical potential  $\mu$  and spin-orbit coupling  $\alpha_R$ . Importantly, the spin-orbit coupling has realized in ultracold atoms by the recent experimental achievement of synthetic gauge field, therefore, our one-dimensional ultra-cold atomic system described by  $H_{1D}$  is a promising platform to find the mysterious Majorana fermions.

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## 1. INTRODUCTION

Since the discovery of the fractional quantum hall state [1], the concept of the topological order, which was first proposed explicitly by Wen [2], has been developed very fast and used in many condensed matter systems. A gapped system, such as the Pfaffian state proposed by Moore and Read [3], which possesses topological order, may have practical use in the topological quantum computation (TQC) on account of its quasi-particles' non-trivial properties, such as non-Abelian statistics, and the tolerant ability to the decoherence from the environment [4].

Read and Green [5] pointed out that the zero energy Majorana fermion modes existing at the cores of a 2D spinless p-wave superconductor in the weak-phase [6, 7] are the same as the nonabelions in the Pfaffian state [3], and they are non-Abelian quasi-particles [8]. Kitaev constructed a toy model and showed that Majorana fermions exist as end states of a spin-polarized 1-D superconductor [9]. This model supplies an insightful way to find the interesting single Majorana fermion. Recently, many groups have proposed different systems to engineer topological superconductivity (TSC) with Majorana fermions as a bonus [10–17]. Among them, papers [14, 15] recognize that the topological superconductivity can be perhaps most easily engineered in one-dimensional semiconducting wires deposited on an s-wave superconductor, and provide the first realistic experimental setting for Kitaev's model and a platform to find and manipulate single Majorana fermion by braiding [18]. The authors of [19] propose that Au wires in proximity to doped LSCO( $L_{2-x}Sr_xCuO_4$ ), a  $(d_{x^2-y^2})$  wave superconductor

can be a more promising candidates for realizing single Majorana fermion.

In addition to fractional quantum hall systems and topological superconductors, topological non-trivial superfluid, which can be deduced from an underlying normal superfluid, e.g., s-wave superfluid, also supports non-Abelian Majorana fermions. With the rapidly developing technology available for the quantum control of ultra-cold atomic systems, clean environment and highly tunable parameters, ultracold atomic systems may serve as an idea platform for the observation of topological superfluidity and topological phase transition. Importantly, the realization of spin-orbit coupling in ultracold atoms by the recent experimental achievement of synthetic gauge field has made a firm step to engineer topological superfluidity and non-Abelian quasi-particles therein [20, 21].

With the introduction of the spin-orbit coupling, the energy gap of superconductors or superfluids with asymmetry pairing wave functions will close at some points in the first Brillouin zone. If the system is under certain symmetries, e.g. time reversal symmetry, particle-hole symmetry etc., and the manifold is not closed, the system will possess robust gapless edge excitations protected by these symmetries. However, once the symmetry protecting the gapless excitations is broken, e.g. DIII class to D in 3D case [22], the gapless excitations will no longer be protected and will be absorbed by some random impurities. Luckily, there are some cases [22] even the symmetry is broken, there will be gapless excitations that can still exist robustly.

In this article, we study the one-dimensional ultracold atomic system with spin-orbit coupling both for the time reversal symmetry and the time-reversal-symmetry-breaking case. We find that there is only one topologically nontrivial superfluid phase, which is always protected by an energy gap away from the normal superfluid, which agrees with the result in Ref.[19]. What most in-

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terests us is that we can reach the topological superfluid (TSF) region explicitly by tuning the parameters according to the phase-separation diagrams and directly probe the single Majorana fermions.

## 2. MODEL STUDY

We consider a one-dimensional ultracold atomic system with spin-orbit coupling and the Hamiltonian is

$$H_{1D} = H_t + H_{SO} + H_I + H_Z, \quad (1)$$

where

$$\begin{aligned} H_t &= -\frac{1}{2}t \sum_{j,\alpha} \left( \psi_{j+1,\alpha}^\dagger \psi_{j,\alpha} + h.c. \right) - \sum_{j,\alpha} \mu \psi_{j,\alpha}^\dagger \psi_{j,\alpha}, \\ H_{SO} &= -\frac{1}{2} \sum_{j,\alpha,\beta} \left( i\alpha_R \psi_{j+1,\alpha}^\dagger (\sigma_y)_{\alpha\beta} \psi_{j,\beta} + h.c. \right), \\ H_I &= \frac{1}{2} \sum_{\alpha\beta} \sum_{ij} g_{ij} \psi_{i,\alpha}^\dagger \psi_{j,\beta}^\dagger \psi_{j,\beta} \psi_{i,\alpha}, \\ H_Z &= \sum_j V_Z \left( \psi_{j,\uparrow}^\dagger \psi_{j,\uparrow} - \psi_{j,\downarrow}^\dagger \psi_{j,\downarrow} \right), \end{aligned} \quad (2)$$

where  $\psi_j$  is a fermion operator at site  $j$ ,  $\alpha$  and  $\beta$  are the spin indices,  $t$  is the hopping amplitude,  $\mu$  is the chemical potential,  $\alpha_R$  is the spin-orbit coupling strength,  $H_I$  is the interaction,  $g_{ij}$  is the interaction strength between site  $i$  and site  $j$ , and is negative in this model,  $\sigma_y$  is a pauli matrix,  $H_Z$  is the Zeeman term which breaks time reversal symmetry, and  $V_Z$  denotes the strength.

By Fourier transformation and mean field approach, the above Hamiltonian will take the form

$$H_{1D} = H_t + H_{SO} + H_{SF} + H_Z, \quad (3)$$

where

$$\begin{aligned} H_t &= -\sum_{k,\alpha} [t \cos(k) + \mu] \psi_{k,\alpha}^\dagger \psi_{k,\alpha}, \\ H_{SO} &= -\sum_k i\alpha_R \sin(k) \left( \psi_{k,\uparrow}^\dagger \psi_{k,\downarrow} - \psi_{k,\downarrow}^\dagger \psi_{k,\uparrow} \right), \\ H_{SF} &= \sum_k \left[ \Delta(k) \psi_{k,\uparrow}^\dagger \psi_{-k,\downarrow}^\dagger \right. \\ &\quad \left. + \Delta^*(k) \psi_{-k,\downarrow} \psi_{k,\uparrow} \right] - |\Delta_0|^2/J, \\ H_Z &= \sum_k V_Z \left( \psi_{k,\uparrow}^\dagger \psi_{k,\uparrow} - \psi_{k,\downarrow}^\dagger \psi_{k,\downarrow} \right), \end{aligned} \quad (4)$$

where  $\Delta(k) = \frac{1}{N} \sum_{k'} g(k-k') \langle \psi_{-k',\downarrow} \psi_{k',\uparrow} \rangle = \Delta_0 \cos(k)$  is the pairing amplitude,  $g(k-k') = g \cos(k-k')$  is the Fourier form of  $g_{ij}$ ,  $g$  is a real constant.  $\Delta_0$  is a complex constant, here we make it real for convenience. Here the pairing we are interested in is the  $d_{x^2-y^2}$  type.  $N$  is the number of sites and the lattice constant  $a$  is set as unit,  $J = \frac{g}{N}$ . Strictly speaking, making mean field approximation is not proper for a one-dimensional system, as fluctuations are strong and there is no true long range order in a homogeneous system with non-zero temperature in the thermodynamic limit, according to the well-known Hohenberg-Mermin-Wagner theorem. However, as shown in Ref.[23], by confining the system in a box with finite length  $L$  or in a harmonic trap to avoid this technology difficulty, the authors found the mean-field methods provide a useful description in the weakly or moderately interaction regimes by comparing the mean-field result with the exact of asymptotically exact Bethe Ansatz solutions. In the following, we will set the length of the system to be  $L = Na = N$  and  $T = 0$

In the momentum space, under the Nambu spinor representation  $\Psi(k)^\dagger = \{\psi_{k,\uparrow}^\dagger, \psi_{-k,\downarrow}^\dagger, \psi_{-k,\downarrow}, \psi_{k,\uparrow}\}$ , the Hamiltonian can be rewritten as

$$H_{1D} = \frac{1}{2} \sum_k \Psi(k)^\dagger H(k) \Psi(k) - \Delta_0^2/J, \quad (5)$$

where

$$\begin{aligned} H(k) &= \begin{bmatrix} h(k) & \Lambda(k) \\ \Lambda(k)^\dagger & -h^T(-k) \end{bmatrix}, \\ h(k) &= \varepsilon_k \sigma_0 + \alpha_R \sin(k) \sigma_y + V_Z \sigma_z, \\ \Lambda(k) &= i\Delta(k) \sigma_y. \end{aligned} \quad (6)$$

where  $\varepsilon_k = -t \cos(k) - \mu$ . After diagonalizing, the Hamiltonian is of the form

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$$H_{1D} = \frac{1}{2} \sum_k \left\{ [(E_1(k) - E_3(k)) \alpha_{k,\uparrow}^\dagger \alpha_{k,\uparrow} + [E_2(k) - E_4(k)] \beta_{k,\downarrow}^\dagger \beta_{k,\downarrow} + E_3(k) + E_4(k)] \right\} - \frac{\Delta_0^2}{J} + \dots, \quad (7)$$

where  $E_1(k), E_2(k), E_3(k), E_4(k)$  are in the order  $\{++, +-, -+, --\}$  of

$$E(k) = \pm \sqrt{\varepsilon_k^2 + \alpha_R^2 \sin^2(k) + \Delta_0^2 \cos^2(k) + V_Z^2} \pm 2\sqrt{\varepsilon_k^2 \alpha_R^2 \sin^2(k) + \varepsilon_k^2 V_Z^2 + V_Z^2 \Delta_0^2 \cos^2(k)}, \quad (8)$$

and ellipsis stands for the terms which are constant and independent of  $\Delta_0$ ,  $\alpha_{k,\uparrow}^\dagger(\beta_{k,\downarrow}^\dagger)$  is the creation operators of the excitation. The ground state is  $|0\rangle$ , which satisfies  $\alpha_{k,\uparrow}(\beta_{k,\downarrow})|0\rangle = 0$ , and the energy of the ground state is

$$E_0 = \frac{1}{2} \sum_k [E_3(k) + E_4(k)] - \frac{\Delta_0^2}{J}, \quad (9)$$

which needs to satisfy the condition of mean-field-approximation assumption

$$\frac{2}{J} = \frac{1}{2} \sum_k \left[ \frac{A + V_Z^2}{E_3(k)A} + \frac{A - V_Z^2}{E_4(k)A} \right] \cos^2(k), \quad (10)$$

where  $A = \sqrt{\varepsilon_k^2 \alpha_R^2 \sin^2(k) + \varepsilon_k^2 V_Z^2 + V_Z^2 \Delta_0^2 \cos^2(k)}$ .

### 3. RESULTS AND DISCUSSIONS

First, for the time-reversal invariant case ( $V_Z = 0$ ), we have

$$E_3(k), E_4(k) = -\sqrt{(\varepsilon_k \pm \alpha_R \sin(k))^2 + \Delta_0^2 \cos^2(k)}. \quad (11)$$

When  $V_Z = 0$ , the Hamiltonian  $H(k)$  possesses a chiral symmetry. Therefore, the Hamiltonian can be brought into an off-diagonal form by a unitary transformation [24]

$$\begin{aligned} \tilde{H} &= V H(k) V^\dagger = \begin{pmatrix} h(k) + i\mathcal{T}\Lambda(k)^\dagger & \\ & h(k) - i\mathcal{T}\Lambda(k)^\dagger \end{pmatrix} \\ &\simeq \begin{pmatrix} Q_k^\dagger & Q_k \end{pmatrix}, \end{aligned} \quad (12)$$

with

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ & -i\mathcal{T} \end{pmatrix}, \quad (13)$$

and  $\mathcal{T} = i\sigma_y$ ,  $Q_k = \frac{1}{2}[e^{i\theta_-(k)}(\sigma_0 - \sigma_y) + e^{i\theta_+(k)}(\sigma_0 + \sigma_y)]$ , where  $e^{i\theta_\pm(k)} = \frac{-t \cos(k) - \mu \pm \alpha_R \sin(k) + i\Delta_0 \cos(k)}{\sqrt{[-t \cos(k) - \mu \pm \alpha_R \sin(k)]^2 + [\Delta_0 \cos(k)]^2}}$ . In Eq.(12), the meaning of  $\simeq$  is the magnitude of the eigenvalues of  $\tilde{H}$  has normalized.

In one dimension, the Fermi-surface topological invariant (FSTI) of a time-reversal-invariant (TRI) superconductor or superfluid is [24]

$$N_{1D} = \prod_s [sgn(\delta_s)], \quad (14)$$

where  $s$  is summed over all the Fermi points between 0 and  $\pi$ . If a system has odd number of Fermi points between 0 and  $\pi$  with negative pairing, in other words,  $N_{1D} = -1$ , the system is non-trivial, otherwise  $N_{1D} = 1$ , and the system is trivial.

In our system, there is only one Fermi point ( $k = \frac{\pi}{2}$  with  $|\mu| = \alpha_R$ ) between 0 and  $\pi$ , so the sign of  $\delta_s$  directly determines the Hamiltonian is trivial or non-trivial. The sign of  $\delta_s$  is positive or negative corresponding to the change in  $\theta_\pm(k)$  across  $k = \frac{\pi}{2}$  is  $-\pi$  or  $\pi$  [24]. Here, in the weak pairing limit, by increasing  $k$  from  $\frac{\pi}{2} - \epsilon$  to  $\frac{\pi}{2} + \epsilon$  with  $\alpha_R \in (|\mu| - \delta, |\mu| + \delta)$  ( $\epsilon, \delta$  are small positive constants), we find, for  $\alpha_R > \mu$  (we only consider  $\mu > 0$  on account of symmetry), the real and imaginary part of  $-t \cos(k) - \mu + \alpha_R \sin k + i\Delta_0 \cos(k)$  change as

$$-\tilde{\delta} + i\tilde{\epsilon} \longrightarrow \tilde{\delta} + i\tilde{\epsilon} \longrightarrow \tilde{\delta} - i\tilde{\epsilon} \quad (15)$$

with

$$\begin{aligned} \theta_+(k) &\longrightarrow \theta_+(k) - \pi \longrightarrow \theta_+(k) + 2\pi, \\ \Delta\theta_+(k) &= +\pi, \end{aligned} \quad (16)$$

for  $\alpha_R < \mu$ ,

$$-\tilde{\delta} + i\tilde{\epsilon} \longrightarrow -\tilde{\delta} - i\tilde{\epsilon} \longrightarrow \tilde{\delta} - i\tilde{\epsilon} \quad (17)$$

with

$$\begin{aligned} \theta_+(k) &\longrightarrow \theta_+(k) - 2\pi \longrightarrow \theta_+(k) + \pi, \\ \Delta\theta_+(k) &= -\pi, \end{aligned} \quad (18)$$

( $\tilde{\delta}, \tilde{\epsilon}$  are small positive real numbers) therefore, when  $\mu < \alpha_R$ ,  $N_{1D} = sgn(\delta_s) = -1$ , the system is non-trivial; when  $\mu > \alpha_R$ ,  $N_{1D} = sgn(\delta_s) = 1$ , the system is trivial.

Base on Eq.(10), Eq.(11) and Eq.(14) and the above analysis, if we fix the strength of spin-orbit coupling, we find that there exists a critical chemical potential  $\mu_c$  where a superfluid-normal state transition takes place, as shown in Fig.1(a). Above the critical value, it's the normal state where the mean-field-approximation assumption is not valid. Below it, the superfluid phase appears and exists only when the chemical potential is not too large. In this region, when  $\mu > \alpha_R$ , the  $Z_2$  invariant  $N_{1D} = 1$  where is the normal superfluid phase region, and when  $\mu < \alpha_R$ ,  $N_{1D} = -1$  where is the topological superfluid phase region, as shown in Fig.1(a). The line  $\mu = \alpha_R$  separating NSF from TSF is a critical line, crossing this line, the topological number changes and the topological phase transition takes place. Because there is no symmetry breaking while crossing the critical line,

the topological order  $N_{1D}$  is the only parameter to distinguish these two phases.

Second, for the time-reversal-symmetry-breaking case ( $V_Z \neq 0$ ), the  $Z_2$  Majorana number  $\mathcal{M}$  of  $H(k)$  is the new topological invariant to determine the system is topologically trivial or topologically non-trivial. Following the Refs.[9, 19, 25] the Majorana number can be obtained by the formula

$$\mathcal{M} = \text{sgn}[PfB(0)]\text{sgn}[PfB(\pi)] = \pm 1, \quad (19)$$

where  $\pm 1$  corresponds to topologically trivial and non-trivial states and the antisymmetric matrix  $B(k)$  is defined as  $B(k) = H_{1D}(\sigma_x \otimes \sigma_0)$ . In terms of the parameters of the Hamiltonian, the Majorana number can be written as

$$\text{sgn}[(t + \mu)^2 - (V_Z^2 - \Delta_0^2)][(-t + \mu)^2 - (V_Z^2 - \Delta_0^2)], \quad (20)$$

so the Majorana number is  $\mathcal{M} = -1$  when  $|\sqrt{V_Z^2 - \Delta_0^2} - t| < |\mu| < \sqrt{V_Z^2 - \Delta_0^2} + t$  and  $\mathcal{M} = 1$  otherwise.

In this case, we find that the phase diagrams will have much more structures than the case discussed above. We find that the pairing amplitude  $\Delta_0$  monotonically decreases with the increasing  $V_Z$ , and when the Zeeman field is strong enough,  $\Delta_0$  will be dramatically suppressed and no longer monotonically increase with  $|J|$ , the attractive interaction strength, as shown in Fig.1(b). This can be explained by the fact that with the Zeeman field increasing, the polarization becomes stronger and stronger, and finally, this will destroy the singlet  $d_{x^2-y^2}$  pairing. We also find that  $\mu$  and  $\alpha_R$  have the same effects on the pairing amplitude as the Zeeman field, as shown in Fig.1(c)(d).

In the following, we will discuss the most interesting part of our work. According to the phase diagram Fig.2(a) and Eq.(20), there is a critical line (red solid line), corresponding to  $\mu = \sqrt{V_Z^2 - \Delta_0^2} + t$  and  $\mu = t - \sqrt{V_Z^2 - \Delta_0^2}$ , that separates TSF from NSF. Here TSF means that single Majorana fermion exists in this region, while NSF means superfluid with Majorana doublets (as  $V_Z < \sqrt{t^2 + \Delta_0^2}$ ), which are no longer topologically protected due to time-reversal symmetry breaking. There is also a critical line (black solid line), corresponding to  $\Delta_0 = 0$ , that separates NSF from N. In Fig.2(a), there is a dashed line. This dashed line corresponds to  $\mu = \sqrt{V_Z^2 + \alpha_R^2}$ . On this line, the energy gap closes, however, from Fig.2(a), we see the Majorana numbers on both sides of this line are equivalent, either both are trivial or both are non-trivial. Therefore, there is no topological phase transition across this line. According to Fig.2(a), we can tune  $\mu/t$  to (0.1, 1.45),  $\alpha_R/t$  to (0, 1) and reach the TSF region, then we can apply the spatially resolved radio-frequency spectroscopy, which would show a well isolated signal at zero energy, to detect the associated single Majorana fermions.

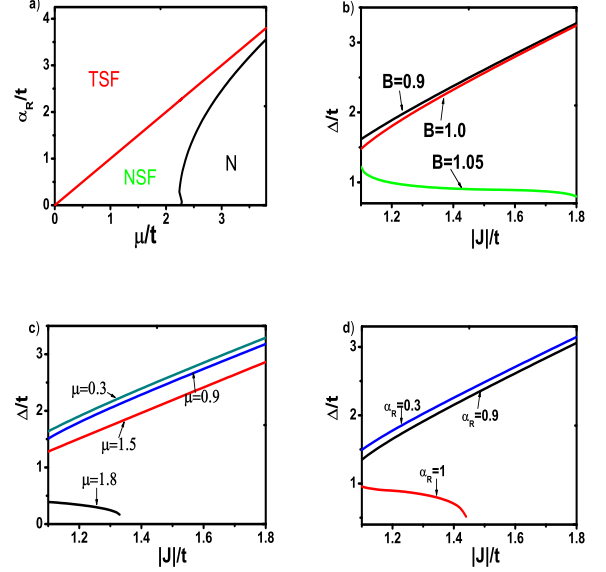


FIG. 1: (Color online) (a) The parameters of  $H_{1D}$  are:  $J = -1.2$ ,  $V_Z = 0$ ,  $t = 1$ , there are two critical lines, which separate the normal superfluid (NSF) from the topological superfluid (TSF) and normal state (N) respectively. (b)  $t = 1$ ,  $\mu = 0.45$ ,  $\alpha_R = 0.3$ , changing the Zeeman field  $V_Z$  from 0.9 to 1.05, we see  $\Delta_0$  monotonically decreases with the increasing  $V_Z$ , and when the Zeeman field is strong enough,  $\Delta_0$  will be dramatically suppressed and no longer monotonically increase with  $J$ , the attractive interaction strength. (c)  $t = 1$ ,  $V_Z = 0.9$ ,  $\alpha_R = 0.3$ ; (d)  $t = 1$ ,  $V_Z = 0.9$ ,  $\mu = 0.45$ , (c), (d) are quite like (b),  $\Delta_0$  also monotonically decreases with the increasing  $\mu$  and  $\alpha_R$ , and there are also critical value  $\mu_c$  and  $\alpha_{Rc}$ , which dramatically suppress  $\Delta_0$ .

In order to obtain the information that how the TSF region changes with other tunable parameters and make single Majorana fermion measurements more achievable, Figs.2(b)-(d) are given. Among them, Fig.2(b) and Fig.2(c) show that by increasing the attractive interaction or reducing the Zeeman field, although the NSF region are broadened, the TSF region are greatly reduced. However, by increasing both the attractive interaction and the Zeeman field, as shown in Fig.2(d), the TSF region are greatly broadened. Such effect will be quite useful in experiments, as the larger the TSF region is, the more detectable the single Majorana fermions are. From Figs.2(a)-(d), we can find TSF tends to form in the high polarization area where the pairing amplitude is small and the imbalance of the chemical potential is large.

#### 4. CONCLUSIONS

In this paper, we propose the one-dimensional Hamiltonian  $H_{1D}$  and discuss it both for the time-reversal-

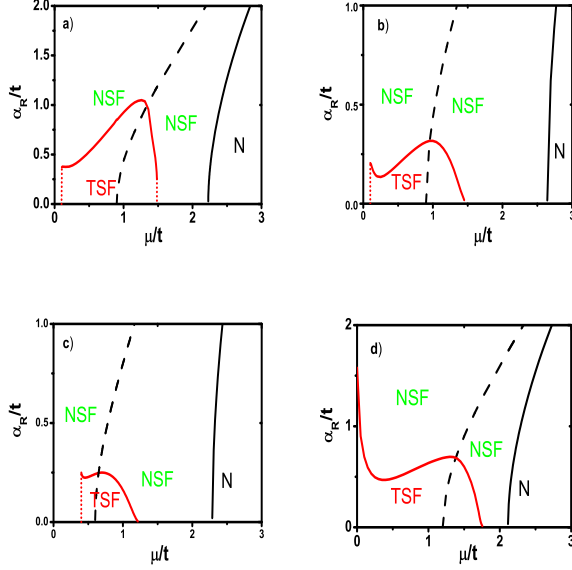


FIG. 2: (Color online) The parameters of the Hamiltonian are: (a)  $t = 1$ ,  $V_Z = 0.9$ ,  $J = -1.2$ ; (b)  $t = 1$ ,  $V_Z = 0.9$ ,  $J = -1.5$ ; (c)  $t = 1$ ,  $V_Z = 0.6$ ,  $J = -1.2$ ; (d)  $t = 1$ ,  $V_Z = 1.2$ ,  $J = -1.6$ . the red solid line is a critical line, separating TSF from NSF, the dotted line is a natural extension of the red solid line, it is also a phase boundary, separating TSF from NSF; the black solid line is also a critical line, separating NSF from N. The dashed line is not a critical line, across it, no phase transition takes place. Comparing (b)(c) with (a), we find that the TSF regions shrink either with  $V_Z$  decreasing or with  $|J|$  increasing. In (d), we fix the ration  $V_Z/J$  to be the same as (a) and keep  $t = 1$ , then we find the TSF region is broadened.

invariant case and the time-reversal-symmetry-breaking case. By numerical solving the self-consistent equation Eq.(10), we obtain different phase-separation diagrams under different conditions. From the diagrams, we find, with the spin-orbit coupling and the Zeeman field, TSF exists. By tuning the parameters, such as  $\mu$  and  $\alpha_R$ , we can reach the TSF region, where single Majorana fermions exist. To detect single Majorana fermions, we can apply the spatially resolved radio-frequency spectroscopy, which would show a well isolated signal at zero energy.

With the rapidly developing technology available for the quantum control and the introduction of spin-orbit coupling to ultra-cold atomic systems, we believe that

our one-dimensional ultra-cold atomic system described by  $H_{1D}$  is a promising platform to find the mysterious Majorana fermions.

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